ISSN 1750-9823 (print)
International Journal of Sports Science and Engineering
Vol. 02 (2008) No. 01, pp. 03-14

Modeling and Computer Simulation of Bow Stabilization in the Vertical Plane

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(*Received October 24 2007, accepted January 10 2008*)

Abstract. The aim of the research is to develop a method of mechanical and mathematical modeling and computer simulation of dynamic stabilization of bow rotation in the vertical plane intending to get practical recommendations for the sport of archery. Behavior of a flexible stabilizer in the main plane of the modern sport bow designed in the frame of International Archery Federation is analyzed using a mechanical and mathematical model. The model is designed basing on Euler-Bernoulli beam and Lagrange equations of the second kind. An engineering oriented method based on virtual modes and Rayleigh-Ritz procedure is developed to study natural frequencies of the archer-bow-stabilizer system. The results of modeling of the archer-bow-arrow system correlate with well-known results of a high-speed video analysis: the process of common motion has a significant non-linear character.

Keywords: sport archery, bow, dynamics, stabilization, modeling, simulation.

1. Introduction

The main features of the modern target bow designed according FITA (International Archery Federation) Standard [3] are a rigid central part named a riser with a handle and two flexible limbs fixed at the ends of the handle (Fig. 1). The unstrung limbs have got recurved horns at the free ends and are bent backward off the internal part of a bow. To make a shot, a bow is situated vertically with its main plane. Despite a simple construction at first sight, bow moves surprisingly complicated. Its motion is in 3D space before, during, and after the shot. Displacement of the bow riser is significantly smaller than displacement of the arrow, string's displacement, and two limbs' displacement in the vertical plane. All the system moves laterally too, but this movement is smaller in comparison to the movement in the main (vertical) plane. To avoid some part of bow riser motion, modern sport bows are supplied with a long cantilever rod (or a multi-rod packet) mounted in front of the riser directly to a target. Additionally, there are from one to three short cantilever rods mounted on the riser in different directions relatively to the main rod but their role in bow stabilization is secondary and their availability on the riser is optional.

Bow riser movement before the shot, i.e. before the string releases off archer's fingers, is under the archer control and directly affects aiming. Movement during the shot, i.e. during string and arrow common motion is partly controlled by the archer hand holding a bow riser and partly is free. This movement affects aiming to some extent. Bow movement after the shot, i.e. after an arrow launches the string does not affects arrow's flight but causes the archer to modify a style to anticipate the movement. Ellison (1996) classified this motion as displacement, rotation, and vibration and qualified the character of them according to the three phases mentioned above [1]. To summarise a stabiliser system's behavior, he pointed two main functions: to maximise the dynamic stabilisation of the bow, particularly with respect to the bow recoil and to establish a 'balanced bow' with the force on the bowhand running through the bow arm. The main part of the handle motion is rotation.

The aim of the research is to develop a method of mechanical and mathematical modeling and computer simulation of dynamic stabilization of bow rotation in the vertical plane intending to get practical recommendations for the sport of archery.

Published by World Academic Press, World Academic Union

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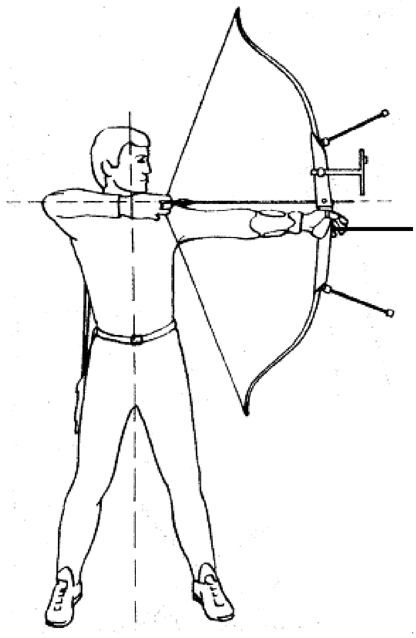

Fig. 1: Archer with a sport bow [3].

2. Modeling of a Stabilizer

2.1. Common Approach to the Model Design

Let's consider a compound hinge and a rode mechanism as a bow scheme model [7]. A stabilizer is modeled as an elastic rod joined to the handle like a cantilever beam. According to the results of the video investigation [2], a bow stabilizer bends according the main mode of natural oscillation. Energetic methods of dynamic mechanics obtain comprehensive accuracy for mechanical engineering calculation. Therefore, we can design a mechanical and mathematical model of a bow with stabilizer using a hypothetical function of the main mode of the beam. According to the theorem of applied mechanics, a precision of natural frequencies obtained with the energetic methods is near the precision of hypothetic functions having been used [4].

Sport archers stretch a bow during the time of common motion of a string and an arrow with the fixed hand trying to keep a steady pose of a body. Body mass is significantly greater than bow mass. Therefore, we can assume the point of contact as immovable, i.e. a pivot point. Angular displacement of the riser (measured in radians) is much smaller than unite; therefore we can use a linear model for its modeling. Inertial properties of the bow are modeled with a load relatively to the pivot point. So, the model of the stabilizer could be a one end pinned elastic beam with a point mass that has got a moment of inertia on the same end.

To approbate hypothetic functions of stabilizer dynamic bend, we consider a range of ratio for beam and joined load mass. If the moment of inertia of the load is much more than the moment of inertia of the beam relatively the pivot point, the problem is near analysis of natural oscillation of a cantilever elastic beam. If the other way rounds, the problem is about the elastic beam with one free end and another pinned end.

At the very beginning, we assume the hypothetic function of the main mode as a function of static bend of a cantilever beam loaded by a concentrated force at the free end (Fig. 2 a):

$$\eta(z) = \frac{Fz^2}{6\varepsilon}(z - 3l), \tag{1}$$

where F is a loading force; ε is distributed bend stiffness of the beam; l is length of the beam; z is longitudinal coordinate; η is transverse displacement. The function (1) satisfies three of four boundary conditions, i.e. the geometrical conditions and one of two dynamic conditions. The geometrical conditions are zero displacement and zero angle of cross-section turn at the fixed end. Only one dynamic condition is satisfied by the function (1), i.e. zero moment of the force, but zero cross-section force at the free end isn't

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satisfied. However, this function is very fruitful in the problem because the error of the main natural frequency isnear 0,73 % [6].

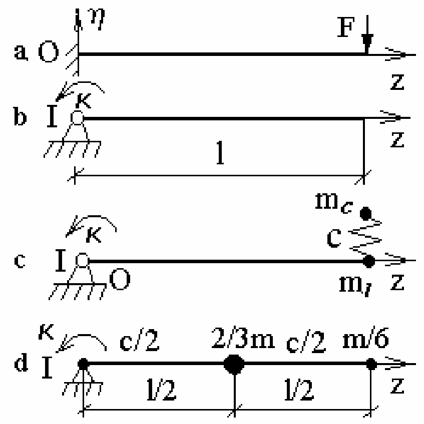


Fig. 2: Scheme models of a bow stabilizer as a cantilever elastic beam (a) and as one-end hinged elastic beam: models 1 and 2 (b); models 3, 4, and 5 (c); the model 6 (d).

As a hypothetic function of the main natural oscillation mode of one hinged beam (Fig. 2 b) we assume a sum of one-half sinusoid wave and a linear function:

$$\eta = A sin\frac{\pi z}{l} + \kappa z , \tag{2}$$

where A is a function of time; κ is an angle of sinusoid wave turn.

Like (1) the function (2) satisfies only three of four boundary conditions, i.e. zero displacement and zero force moment at the pinned end of the beam and one dynamic of the two boundary conditions, i.e. zero force moment at the free end. Another dynamic boundary condition is not satisfied, i.e. zero cross-section force at the free end. Despite this, the function (2) allows an appreciated precision of the main natural frequency because the error is near 1,10 % [10].

Because there are no results on the problem of natural frequencies of a one end pinned beam with a load in well-known mechanical and mathematical publications, we consider this problem using Hamilton variation principle:

$$\delta\int_{t_1}^{t_2}(T-P)dt = 0 ,$$

where $T = \frac{1}{2}\left[\int_0^l \mu\left(\frac{\partial\eta}{\partial t}\right)^2 dz + I\left(\frac{\partial^2\eta}{\partial z\partial t}\right)^2_{z=0}\right]$ and $P = \frac{1}{2}\int_0^l \varepsilon\left(\frac{\partial^2\eta}{\partial z^2}\right)^2 dz$ are kinetic and potential energy correspondingly; μ is distributed mass of the beam; t is time; I is moment of inertia of the load relatively the hinge axis (see Fig. 2 b). Placing the two last expressions of energy in the Hamilton functional, we get correspondent differential equation:

$$\mu\frac{\partial^2\eta}{\partial t^2} + \varepsilon\frac{\partial^4\eta}{\partial z^4} = 0 ;$$

and boundary conditions

$$z = 0, \eta = 0, \varepsilon\frac{\partial^2\eta}{\partial z^2} = I\frac{\partial^3\eta}{\partial z\partial t^2} ; \qquad z = l, \frac{\partial^2\eta}{\partial z^2} = 0, \frac{\partial^3\eta}{\partial z^2} = 0 . \tag{3}$$

Solutions of the problem (3) obtained using Krylov functions are roots of the determinant:

$$\begin{vmatrix} 2kl & v(kl)^4 & v(kl)^4 \\ ch(kl) + cos(kl) & sh(kl) & -sin(kl) \\ sh(kl) - sin(kl) & ch(kl) & -cos(kl) \end{vmatrix} = 0 , \tag{4}$$

where $kl = \sqrt[4]{\frac{ml^3\omega^2}{\varepsilon}}$ are dimensionless values of natural frequencies; m is mass of the beam; ω is circular natural frequencies; $v = \frac{I}{ml^2}$ is dimensionless value of the moment of inertia of the load. Zero solution of the equation (4) corresponds with common rotation of the beam and the load relatively the hinge axis.

As v=0, we get kl=0; 3,927; 7,069; 10,210; 13,352; …, which are the same as well-known solutions for the beam with one hinged end $\frac{\pi(4i-3)}{4}$, where i is a number of natural frequency [5]. There is no zero solution as $v=\infty$: kl=1,875; 4,694; 7,855; 10,996; …, which are the same as well-known solutions for the cantilever beam $\frac{\pi(2i-1)}{2}$. Solutions of the main frequency for different relationship of mass-inertial parameters are presented on the Fig. 3 (ODE line).

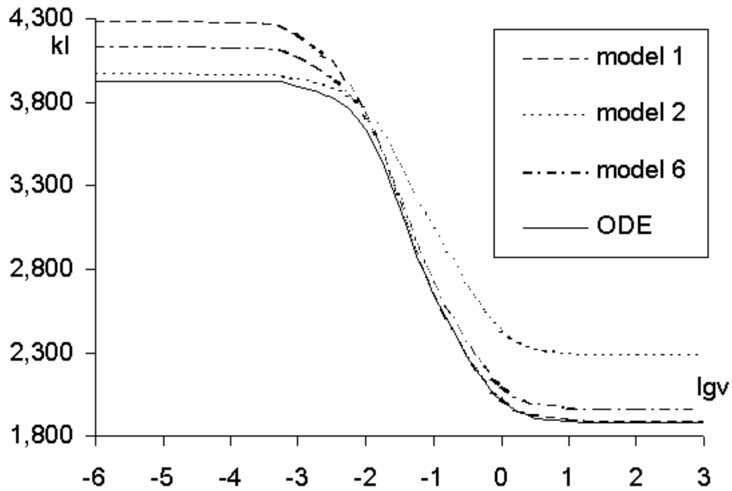


Fig. 3: Main natural frequency vs. bow and stabilizer mass-inertial parameters: ODE is considered as an exact solution (eq. 4).

2.2. Model Versions of the Stabilizer

Using a hypothetical function of the main natural mode, we can apply Lagrange equations of the second kind:

$$\frac{d}{dt}\left(\frac{\partial T}{\partial q_i'}\right) - \frac{\partial T}{\partial q_i} + \frac{\partial P}{\partial q_i} = 0, \tag{5}$$

where q_i are generalized coordinates; prefix shows a partial derivation in time, i.e. $(\,)' \equiv \partial / \partial t$.

<u>Model version 1</u> is designed as a sum of (1) and a linear function:

$$\eta = A\left(\frac{z}{l}\right)^2\left(3 - \frac{z}{l}\right) + \kappa z \tag{6}$$

As generalized coordinates, there are A and κ. The angle of the load turning is $\left(\frac{\partial \eta}{\partial z}\right)_{z=0} = \kappa$. After substituting of the hypothetical function (6) in the equations of energies and then in (5), we get expressions for the main natural frequency:

$$(kl)^4 = 12 / \left(33/35 - \frac{121/400}{1/3 + v}\right), \tag{7}$$

where (/) is a sign of division.

For the <u>model version 2</u>, we applied the hypothetical function (2). Like has been made before, we get the angle of the load's turn that is presented here as expression $\left(\frac{\partial \eta}{\partial z}\right)_{z=0} = \kappa + \frac{\pi A}{l}$. Correspondent expression of the main frequency is:

$$(kl)^4 = \frac{\pi^4}{2} / \left[\frac{1}{2} + \nu\pi^2 - \frac{(1/\pi + \nu\pi)^2}{1/3 + \nu}\right]. \tag{8}$$

In the <u>model version 3</u>, hypothetical functions are assumed with the expressions:

$$\eta_l = \kappa l; \;\; \eta_c = \kappa l + A. \tag{9}$$

Virtual stiffness and mass of the beam are located at the free end (Fig. 2 c). Using the function (1), we get: $c = \frac{3\varepsilon}{l^3}$; $m_c = \frac{33}{140} m$. The angle of the load turning is $\left(\frac{\partial \eta}{\partial z}\right)_{z=0} = \kappa$. Correspondent expression for the main frequency is:

$$(kl)^4 = \frac{140(1/3 + \nu)}{11\nu} \tag{10}$$

<u>Model version 4</u> is different of the model version 3 only with the value of virtual mass $\left(m_c = \frac{m}{3}\right)$ that corresponds with the value of the moment of inertia of the beam relatively the hinge axis. Here, a formula for the main natural frequency is:

$$(kl)^4 = \frac{9(1/3 + \nu)}{\nu}. \tag{11}$$

<u>Model version 5</u> is a combination of two previous model versions, i.e. 3 and 4: $m_c = \frac{33}{140} m$; $m_l = \frac{41}{420} m$. General virtual mass $\left(m_c + m_l = \frac{m}{3}\right)$ corresponds with the value of the beam moment of inertia. The main natural frequency is calculated with a formula:

$$(kl)^4 = 3 / \left(33/140 - \frac{(33/140)^2}{1/3 + \nu}\right). \tag{12}$$

In the <u>model version 6</u> the beam mass is located in three points, i.e. at the beam-ends $\left(m_0 = m_2 = \frac{1}{6} m\right)$ and in the middle of the beam $\left(m_1 = \frac{2}{3} m\right)$. Stiffness is concentrated in two points, i.e. in the middle of the beam and at the free end: $c_1 = c_2 = \frac{6\varepsilon}{l^3}$ (Fig. 2 d). This is equal to the stiffness of the cantilever beam: $c = \frac{3\varepsilon}{l^3}$. Using the function (6), we get an expression for the main natural frequency:

$$(kl)^4 = (219/16) / \left(89/96 - \frac{169/576}{1/3 + \nu}\right). \tag{13}$$

The results on the relative accuracy for the all six model versions (7), (9)-(13) relatively the ODE solution (4) are grouped in the Table 1. From the practical point of view, appreciated results regards the accuracy of the main natural frequency is obtained with the model versions 1, 2, and 6. But the best accuracy in a wide range of mass-inertial parameters of the beam and the load $(-1{,}5 < \lg \nu < \infty)$, we get using model version 1. Model version 2 is appreciated only for small loads $(-\infty < lg\,\nu < -3)$. Using the model version 6, we get mediocre level of accuracy, and only in a narrow range of relationship $(lg\,\nu \approx -2)$ the accuracy is appreciated. The main natural frequency results obtained with the model versions 1, 2, and 6 and the ODE solutions (4) are presented in a non-dimensional form (Fig. 3).

Taking into account a real relationship between bow and stabilizer mass-inertial parameters

$(-1{,}5 < lg\,v < 2{,}0)$ and considering the results of calculations (see Table 1 and Fig. 3), we can choose the model version 1 as the best one. The smallest error of the main natural frequency result for this model is 0,16 % when the relationship *v*=0,0537 (*lgv*=-1,27).

Table 1: Related errors of the main frequency for different model versions of a bow and stabilizer system (%)

lgv	Number of a model version					
	1	2	3	4	5	6
-6	9,30	1,10	1055,69	959,79	-34,62	5,26
-5	9,30	1,11	550,01	496,07	-34,61	5,27
-4	9,18	1,12	265,79	235,44	-34,58	5,20
-3	8,10	1,25	107,21	90,01	-34,25	4,61
-2	2,54	3,41	25,59	15,17	-30,66	1,82
-1	0,27	15,52	3,27	-5,30	-12,90	3,08
0	0,66	21,01	0,98	-7,40	-1,35	4,36
1	0,72	21,72	0,76	-7,60	0,51	4,52
2	0,73	21,80	0,71	-7,65	0,69	4,52
3	0,73	21,81	0,74	-7,62	0,74	4,55

3. Approbation of the Model

3.1. Model of Aiming

In a static problem of the bow and stabilizer system [11], we can assume only two external forces, which act to the handle and to the string, i.e. the forces in the points corresponding O (the pivot point) and A (nock point). A mathematical model of the bow at the drawn situation is (Fig. 4):

$$\begin{gathered}\xi_A = l_U \sin\theta_U + s_U \sin\gamma_U;\;\; \xi_A = l_L \sin\theta_L + s_L \sin\gamma_L;\\ \eta_A = h_U + l_U \cos\theta_U - s_U \cos\gamma_U;\; c_U(\theta_U+\varphi_U) = F_U l_U \sin(\theta_U+\gamma_U);\\ \eta_A = s_L \cos\gamma_L - l_L\cos\theta_L - h_L;\;\; c_L(\theta_L+\varphi_L) = F_L l_L \sin(\theta_L+\gamma_L);\\ F_\xi = -F_U \sin\gamma_U - F_L\sin\gamma_L;\;\; F_\eta = F_U\cos\gamma_U - F_L\cos\gamma_L;\\ F_U = f\,\frac{s_U - S_U}{S_U};\;\; F_L = f\,\frac{s_L - S_L}{S_L};\;\; tg\phi = \frac{F_\eta}{F_\xi};\;\; tg\phi = \frac{\eta_A}{\xi_A},\end{gathered} \tag{14}$$

where l_U, l_L are limbs' lengths; s_U, s_L are lengths of string branches in the drawn bow situation; S_U, S_L are lengths of the branches of a free string; h_U, h_L are virtual lengths of the riser, i.e. the distances from the pivot point (O) to the points of virtual elastic elements of limbs; c_U, c_L are stiffness of the limbs; F_U, F_L are forces of the string branches; F_ξ, F_η are projections of the drawn force to the axis of co-ordinates; l_a is a length of an arrow (and drawn distance too); f is a parameter of stiffness of a string. The subdivides «U» and «L» mark corresponding the upper and lower limbs.

Mathematical model of the braced bow is:

$$l_U\cos\theta_{UB} + l_L\cos\theta_{LB} + h_U + h_L = 2s_B\cos\gamma_B;\;\; l_U\sin\theta_{UB} - l_L\sin\theta_{LB} = 2s_B\sin\gamma_B;$$

$$F_B l_U \sin(\theta_{UB}-\gamma_B) = c_U(\theta_{UB}+\varphi_U);\; F_B l_L\sin(\theta_{LB}+\gamma_B) = c_L(\theta_{LB}+\varphi_L);\; F_B = f\,\frac{s_B - S}{S}, \tag{15}$$

where s_B is the string length in the braced situation; F_B is the force of the string stretch; S is the length of a free string (see Fig. 4). Subdivide «B» means parameters of a bow with a braced string.

Systems of equations (14) and (15) include transcendent and non-linear functions, so they could not be solved analytically. Therefore, considering the static problem, we applied numerical method using a program

Find from the software package Mathcad 2000i Professional (www.mathsoft.com).

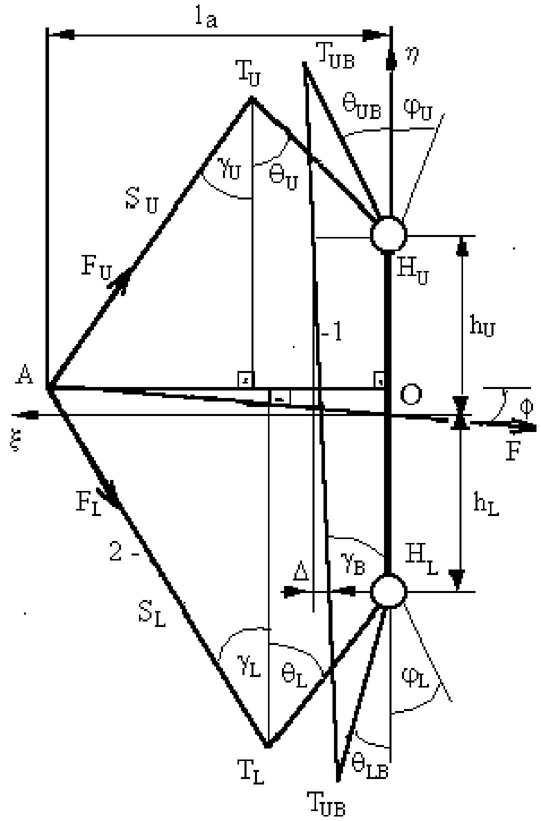


Fig. 4: Static scheme model: braced bow (1); drawn bow (2).

3.2. Bow, Stabilizer, and Arrow Common Motion

The main attempt to the dynamic problem on bow and arrow system has been founded [8, 9]. Using co-ordinates of the nock point, limb angles, the angular co-ordinate of the riser, and the angular co-ordinate of the arrow we could form expressions of energy (Fig. 5):

$$T=\frac{1}{2}\left\{\begin{aligned}&(m_A+m_a)\xi_A'^2+m_A\eta_A'^2+\int_0^{l_a}\mu(z)(\eta_A'+z\psi')^2dz+\left(I_H+m_Uh_U^2+m_Lh_L^2\right)\kappa'^2\\&+I_U(\theta_U'+\kappa')^2+I_L(\theta_L'-\kappa')^2+2\kappa'\begin{bmatrix}m_Ur_Uh_U(\theta_U'+\kappa')\cos(\theta_U+\kappa)-\\-m_Lr_Lh_L(\theta_L'-\kappa')\cos(\theta_L-\kappa)\end{bmatrix}\end{aligned}\right\};$$

$$P=\frac{1}{2}\left[\begin{aligned}&c_U(\theta_U+\varphi_U)^2+c_L(\theta_L+\varphi_L)^2+\frac{f}{S_U}(s_U-S_U)^2+\frac{f}{S_L}(s_L-S_L)^2\\&+\xi_A''\int_0^{l_a}\mu(z)z\psi^2dz+2g\int_0^{l_a}\mu(z)(\eta_A+z\psi)dz\end{aligned}\right] \qquad (16)$$

where $m_A=\frac{1}{3}m_s$ is mass of a string pinned to the nock point; m_s is mass of a string; m_a is mass of an arrow; $\mu(z)$ is distributed mass of an arrow's shaft; z is a co-ordinate fixed to the arrow axis; ψ is an attitude angle of an arrow; I_H is moment of inertia of a riser relatively the pivot point; m_U, m_L are mass of limbs with added mass of string $\left(\frac{1}{3}m_s\right)$ pinned to their nock points; I_U, I_L are moments of inertia of limbs relatively their joints to the riser with addition the same part of string mass; r_U, r_L are distances of centers of mass of limbs to their joints; g is gravity constant.

Bow and arrow interaction has been described according the model (6) taking into account only the interaction at the nock point. The initial position of the arrow relatively the bow in its main (vertical) plane is determined with a rest that holds an arrowhead. A rest is fixed to the handle and has got ability to turn and to disappear just an arrow starts movement. Thanks a small size and mass, the rest does not accumulate significant amount of energy, therefore its interaction with an arrow is not taken into account in the frame of the model.

Solving the dynamic problem like the static problem (14) and (15), we do not consider gravity forces acted the bow. But we take into consideration an arrow weight because its force moment acted the arrow is the same value as the moment of inertial forces.

Kinetic and potential energy of the stabilizer with the main natural form (6) according the model version 1 is:

$$T_{st} = \frac{1}{2} m_{st} \left(\frac{33}{35} A'^2 + \frac{1}{3} l_{st}^2 \kappa'^2 + 2 \frac{11}{20} l_{st} A' \kappa' \right); \; P = \frac{1}{2} c_{st} (2A)^2 , \tag{17}$$

where $c_{st} = 3 \frac{\varepsilon}{l_{st}^3}$ is bend stiffness of a cantilever stabilizer loaded by a transverse force at the free end.

Placing the expressions (16) and (17) to the Lagrange equations (5), we get a system of differential equations of the second power relatively the generated co-ordinates $q_i \equiv \xi_A, \eta_A, \psi, \theta_U, \theta_L, \kappa, A$:

$$\begin{gathered}
(m_A + m_a)\xi_A'' - e_U S_2 + e_L S_4 = 0; \\
(m_A + m_a)\eta_A'' + m_a r_A \psi'' + m_a g - e_U S_1 + e_L S_3 = 0; \\
I_A \psi'' + m_a r_A (\eta_A'' + \xi_A'' \psi + g) = 0 ; \\
I_U (\theta_U'' + \kappa'') + m_U r_U h_U b_1 \kappa'' + c_U (\theta_U + \varphi_U) + e_U l_U (b_1 S_2 - b_2 S_1) = 0 ; \\
I_L (\theta_L'' - \kappa'') - m_L r_L h_L b_3 \kappa'' + c_L (\theta_L + \varphi_L) - e_L l_L (b_3 S_4 + b_4 S_3) = 0 ; \\
\left(I_H + I_U + I_L + m_U h_U^2 + m_L h_L^2 + \tfrac{1}{3} m_{st} l_{st}^2 \right) \kappa'' + m_U r_U h_U \left[b_1 (\theta_U'' + 2\kappa'') - b_2 (\theta_U' + \kappa')^2 \right] \\
+ I_U \theta_U'' - I_L \theta_L'' - m_L r_L h_L \left[b_3 (\theta_L'' - 2\kappa'') - b_4 (\theta_L' - \kappa')^2 \right] + \tfrac{11}{20} m_{st} l_{st} A'' \\
+ e_U \left[S_2 (b_1 l_U + h_U) - S_1 b_2 l_U \right] + e_L \left[S_4 (b_3 l_L + h_L) + S_3 b_4 l_L \right] = 0; \\
m_{st} \left(\tfrac{11}{20} l_{st} \kappa'' + \tfrac{33}{35} A'' \right) + 4 c_{st} A = 0 ,
\end{gathered} \tag{18}$$

where I_A is a moment of inertia of the arrow relatively its tail; r_A is a distance from the tail to the center of mass of the arrow;

$$e_U = \frac{f(s_U - S_U)}{s_U S_U}; \; e_L = \frac{f(s_L - S_L)}{s_L S_L}; \; s_U = \sqrt{S_1^2 + S_2^2}; \; s_L = \sqrt{S_3^2 + S_4^2} ;$$

$$S_1 = h_U + l_U b_1 - \eta_A; \; S_2 = h_U \kappa + l_U b_2 - \xi_A; \; S_3 = h_L + l_L b_3 + \eta_A; \; S_4 = h_L \kappa - l_L b_4 + \xi_A;$$

$$b_1 = \cos(\theta_U + \kappa); \; b_2 = \sin(\theta_U + \kappa); \; b_3 = \cos(\theta_L - \kappa); \; b_4 = \sin(\theta_L - \kappa).$$

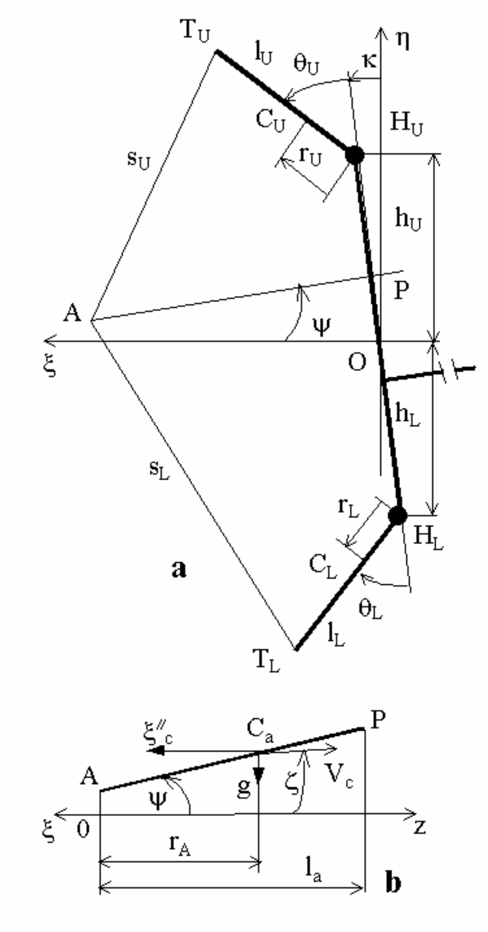


Fig. 5: Dynamic scheme model (a); scheme model of an arrow (b).

The initial conditions of the problem are:

$$t=0,\ \xi_A=l_a;\ \eta_A=\eta_{A0};\ \theta_U=\theta_{U0};\ \theta_L=\theta_{L0};\ \kappa=0;\ \psi=\psi_0;$$
$$A=0;\ \xi'_A=0;\ \eta'_A=0;\ \theta'_U=0;\ \theta'_L=0;\ \kappa'=0;\ \psi'=0\ \ A'=0, \tag{19}$$

where constants η_{A0}, θ_{U0}, θ_{L0} are the solutions of the static problem (14). Zero values of derivations correspond the manner of the sport archer technique, i.e. a breathing is stopped and a pose is motionless. Stabilizer parameters play in the two last differential equations (18).

The system of equations (18) with initial conditions (19) represents a Cauchy problem for the ordinary differential equations of the second power. It is impossible to get analytical solutions for the problem, therefore we used Runge-Kutta method applied in the program NDSolve from the package Mathematica 5.1 (www.wolfram.com).

4. Computer Simulation

Lets consider a modern sport bow with parameters: $l_U=l_L=$ 48 cm; $m_U=m_L=$ 95,3 g; $I_U=I_L=$ 63,44 kgcm^2; $r_U=r_L=$ 21,7 cm; $c_U=c_L=$ 12078 Ncm; $\varphi_U=$ -0,06; $\varphi_L=$ 0,06; $h_U=h_L=$43,4 cm; $I_H=$2710 kgcm^2; $S_U=$80 cm; $S_L=$90 cm; $f=$11900 N; $m_s=$7 g; $l_a=$70 cm; $m_a=$ 25 g; $I_A=$ 55,5 kgcm^2; $r_A=$41,3 cm; l_{st}=1,486 m; m_{st}=0,304 kg; c_{st}=702 N/m. From the static problem (14), we have got $\eta_{A0}=$ 24 mm; $\theta_{U0}=$ 0,826; $\theta_{L0}=$ 0,694. The rest point situates at the coordinate $\eta_{P0}=34$ mm.

The results of solution of the problem for the parameters above are presented in the graphs (Fig. 6, 7). An arrow launches a string nock point as their common longitudinal acceleration becomes zero. At the instant an arrow has the maximal longitudinal speed. The time of bow and arrow common motion is about 15,8 ms. The graphs describe a process of bow stabilization in the vertical plane during the bow and the arrow move together. Bow riser angular motion clockwise is partly compensating by stabilizer bend counterclockwise. A monotone character of these movements testifies a below resonance regime of the process.

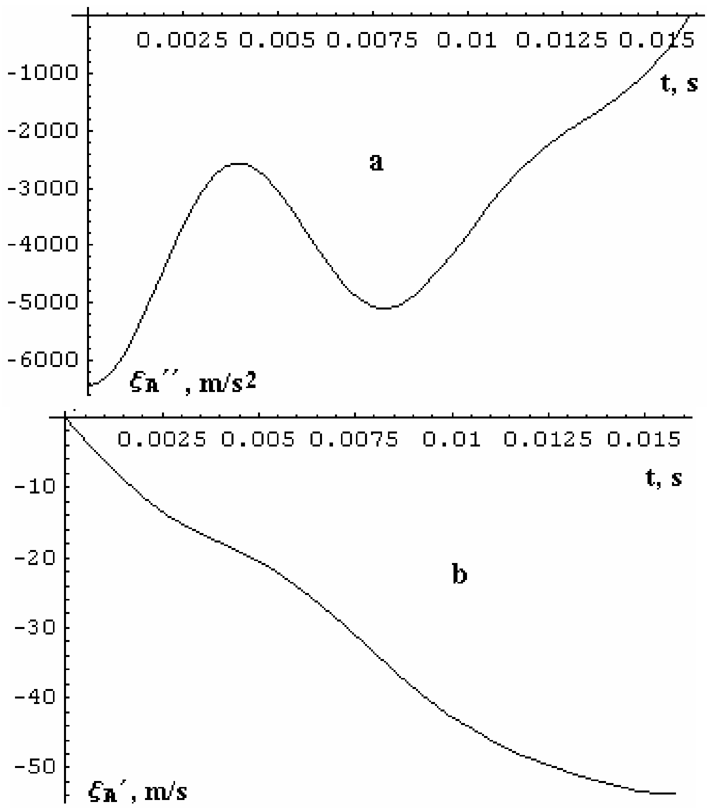
Bow and arrow common motion has an extremely non-linear character. During the motion, the acceleration of the longitudinal displacement ξ_A'' decreases from approximately $640\,g$ to $270\,g$, and then increases again up to $520\,g$ (see Fig. 6 a).; in the finishing phase it decreases to zero. Simultaneously, the longitudinal projection of the arrow velocity increases (non-linear again) up to 52.8 m/s (see Fig. 6 b). At the time, the riser turns on a small angle $\kappa = -0.0006$ rad (see Fig. 6 c), i.e. it moves forward with its upper part. This is proved correct by high speed video film [2].

String and arrow common motion (internal ballistics) is accompanied with intensive oscillations, which are caused by destruction of the static balance of forces at the point of string release. There are seven full cycles of oscillation during the motion (see Fig. 7).

The level and the character of dynamic stabilization are to a certain degree depended on stabilizer beam stiffness. Results of the calculation experiment on the issue are presented in Table 2. We can notice that stabilizer beam stiffness increase causes bow riser turn increase. Comparing the bow without a stabilizer (κ= 0,00191 rad) and the bow with absolutely solid stabilizer (κ_∞= 0,00116 rad), we can determine a difference of a bow riser turn about 65 %.

Table 2: Kinematics parameters of bow stabilization at the instant as an arrow launches a string

c_{st}, N/m	-1000*κ	2*A, mm	100* κ/κ_∞, %
∞	1,16	0	100
2000	1,45	1,32	125
1500	1,53	1,67	132
1000	1,62	2,16	140
500	1,73	2,62	149
0	1,91	-	165



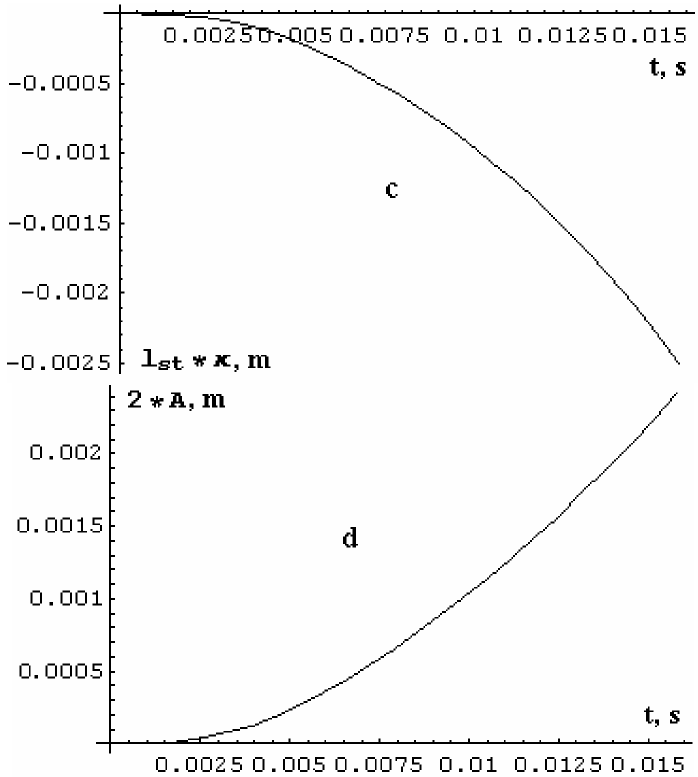


Fig. 6: Kinematical parameters of the system: longitudinal acceleration of the arrow vs. time (a); longitudinal speed of the arrow (b); bow riser angle multiplied by stabilizer length (c); pure bend displacement of the free end of the stabilizer (d).

On the other hand, dynamic bend of the beam stabilizer decreases as the stiffness increases. For example, in the case of a cylindrical tube stabilizer (c_{st}=1000 N/m), the bend is 1,6 times grater than in the case of four rods packet of the equal mass (c_{st}=2000 N/m). At the same conditions, stabilizer mass increase causes decrease of bow turn motion below the resonance zone.

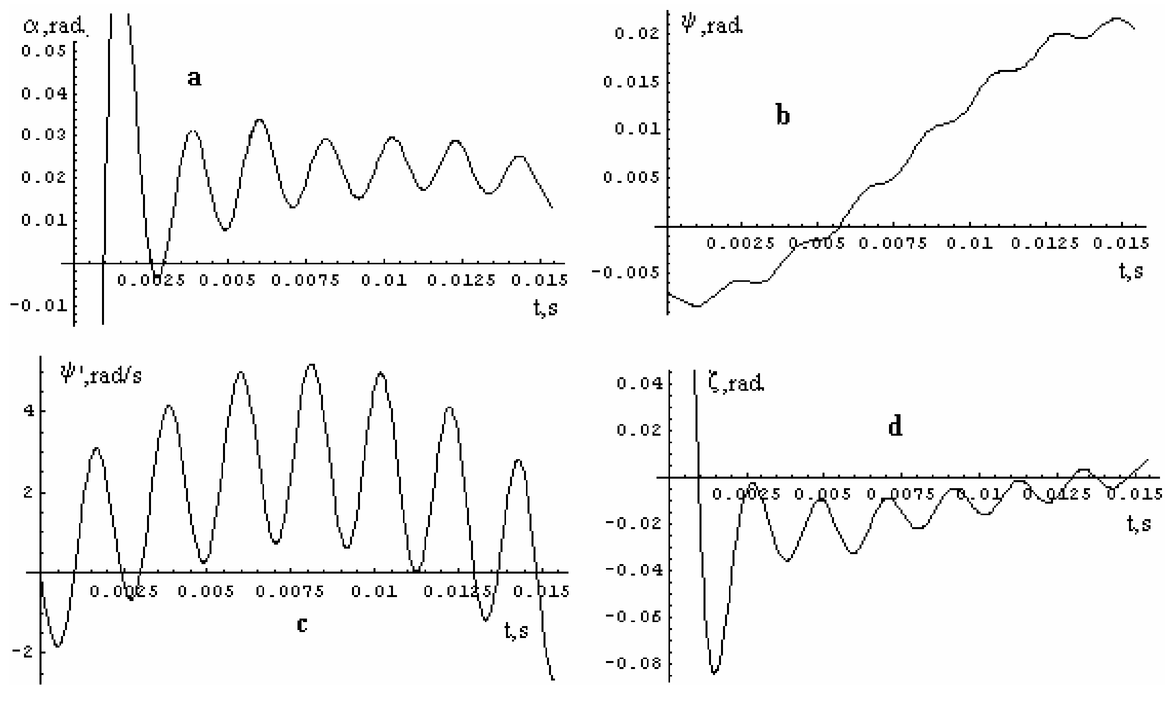


Fig. 7: Kinematical parameters of internal ballistics: an angle of attack of the arrow vs. time (a); an attitude angle (b); an angular speed of the arrow in the vertical plane (c); an angle of the speed vector of the arrow longitudinal movement relatively horizon (d).

5. Conclusions

- For real relationship of bow and stabilizer mass-inertial parameters, we can get the best accuracy of the main natural frequency of the system (near 1,1 %) using a hypothetical function as a combination of a linear function and the function of static bend of a cantilever beam loaded by a force at the free end.
- String and arrow common motion (internal ballistics) is accompanied with intensive oscillations, which are caused by destruction of the static balance of forces at the point of string release. There are seven full cycles of oscillation during the motion.
- Beam stabilizer stiffness increase cause a significant decrease of bow turn motion and decrease of dynamic stabilizer bend. At the same conditions, stabilizer mass increase causes decrease of bow turn motion in below the resonance zone.
- The model describes the process of bow stabilization in the vertical plane during the bow and the arrow move together. Bow riser angular motion clockwise is partly compensating by stabilizer bend counterclockwise. A monotone character of these movements testifies a below resonance regime of the process.
- The attempt to the problem of sport bow stabilization in the vertical plane proposed in the paper is addressed to practical needs of applied engineering mechanics and the archery sport. The models and methods have been adapted for realization in an engineering method using well-known mathematical CAD systems. Numerical results of computer simulation are presented in tabular and graphical form, which makes it easy for sportsmen and coaches to use.

6. References

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7. Appendixes (omitted)

7.1. MathCAD program on the model of archery bow parameters in the drawn situation

7.2. Mathematica computer program on bow stabilization in the vertical plane